

# An Origami project to experiment on mathematical logic

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## Abstract

It is well known that origami is a very good tool to teach mathematics, not only in primary and middle schools, but also at high school and university. For undergraduate students some projects appeared in the past years (among all, see T.Hull, *Project Origami*, A K Peters/CRC Press, 2012) involving many beautiful mathematical subjects.

The authors of this paper often use new origami ideas to teach mathematical contents to high school and undergraduate students—for example, in [M.L.Spreafico, *Activities in Mathematics course for undergraduate students: from origami to software*, in Edulearn17 Proceedings, IATED Ed., 2017], a proposal for visualizing vectors operations and relative position of planes and lines is described, in [C. Cumino, M.L.Spreafico, *Teaching arithmetic through geometry: Origami Pythagorean Tree, Natural Numberø Power, Sum and Series*, pp. 404-415, in Aplimat2017 Proceeding, Vydavatel'stvo Spektrum STU Ed., 2017], the authors propose an origami lesson for series and fractals., while in [S. Serre, *GeoGebra e Origami: una barchetta per navigare tra ipotesi e tesi*, pp. 237-242, in Atti del IV Geogebra Italian Day-2014, Ledizioni Ed.] GeoGebra allows to construct the crease pattern of a simple boat and to investigate changes in the final model starting from different triangular sheet sizes.

In this paper, we describe some lessons in which we use origami to explore and explain the logical structure of mathematical statements. In fact, in our experience, many students have difficulties in reading and writing mathematical statements, fully appreciating their logical structure. In particular they have problems in clearly understanding hypotheses and theses when reading a theorem, in formulating conjectures, or in correctly using necessary and sufficient conditions. Of course, these problems imply that students have difficulties in understanding if an example supports or invalidates a conjecture.

Many times these difficulties have their origin in the abstractedness of mathematical subjects. Our approach is then to use the concreteness of origami to design lessons on logic. This approach allows students to work on propositional calculus experimenting by hand their conjectures and visualizing proofs and examples.

In particular, we choose the origami flatness problem for many reasons: first of all students do not know this problem, in general, and so they all start from the same level=secondly, some local results, such as Maekawa Theorem or Justin Theorem, involve only elementary mathematics (see [O'Rourke, *How to fold it*, Cambridge Univ. Press, 2011], for an overview)=finally, origami allows us to apply the philosophy of "learning by doing" in sentential logic, a very abstract mathematical branch, and to play with mathematics as an experimental science.

We now describe the structure of our lessons.

- Step 1. Motivate students.

We let students watch some videos about technological applications of flat origami. These applications include solar lens, the bullet-proof origami shield designed by BYU mechanical engineers, or the MIT origami robot. In this way students appreciate the great potential for applications of theoretical investigation on flat origami.

- Step 2. Fold flat origami.

Students fold paper sheets in a flat way, unfold the paper and observe the result. We let them notice that if the model doesn't have any internal vertex, it can always be folded flat. Then we consider the first nontrivial case: the flatness in the neighborhood of a single vertex. We guide students, inviting them to control at first the number of the mountain/valley folds and then the angles around the vertex.

- Step 3. Statements.

Students are invited to extrapolate conjectures from their examples. They have to make clear the logical structure of their statements, in particular to formulate both hypotheses and thesis. Then, they rephrase their statements as sufficient and/or necessary conditions. Finally, everyone tries to give a proof or to give a counterexample. We remark that we are interested in analyzing the logical structure of statements. Often it happens that students rediscover known results, as Kawasaki, Maekawa, Justin theorems. In these cases, after giving the correct attribution, we only sketch the proof.

- Step 4. Comparison with known results.

During the lesson we suggest students to compare the logical structure of the proposed results on flatness with the one of some well-known results in calculus that students learn at school such as necessary or sufficient conditions, equivalent conditions, theorems whose proof is constructive, and theorems whose proof is not.

- Step 5. Local vs Global.

We discuss with students the fact that in mathematics, local and global problems have different approaches and results. This holds in the case of origami flatness, too.

- Step 6. Design a new flat origami.

In this last step, we come back to the motivations, i.e. the technological applications of flat origami. We ask students to propose new applications either by creating flat models or by using well-known flat models, as the triangular base.

Last year, we experimented these lessons in an Italian high school, S.I.E.S. - A. Spinelli, Torino, within the ministerial project PLS aimed to increase the number of students enrolled in scientific faculties. Students appreciated both these lessons and flat origami and this approach encouraged them in a more careful reading of sentences during the curricular lessons.

Moreover, both the authors participated and organized training for teachers about this topic.